

Catalysis of Black Holes/Wormholes Formation in High Energy Collisions*

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Abstract

We discuss various mechanisms of catalysis of black holes/wormholes (BH/WH) formation in particles collisions. The current paradigm suggests that BH/WH formation in particles collisions will happen when center of mass energies of colliding particles is sufficiently above the Planck scale (the transplanckian region).

To estimate the BH/WH production we use the classical geometrical cross section. We confirm the classical geometrical cross section of the BH production reconsidering the process of two transplanckian particles collision in the rest frame of one of incident particles. This consideration permits to use the standard Thorne's hoop conjecture for a matter compressed into a region to prove a variant of the conjecture dealing with a total amount of compressed energy in the case of colliding particles.

We calculate geometrical cross sections for different processes and for different background, in particular, for (A)dS. We show that results are in agreement with closed trapped surface (CTS) estimations though there are no general theorems providing that the BH formation follows from CTS's formation.

We show that the process of BH formation is catalyzed by the negative cosmological constant and by a particular scalar matter, namely dilaton, while it is relaxed by the positive cosmological constant and at a critical value just turns off. Also we note that the cross section is sensible to the compactification of extra dimensions and to the particular brane model.

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1 Introduction

Gravity does not play a role in the usual high energy terrestrial physics. However, in the TeV gravity scenario [1] the processes with energy about few TeV become transplanckian and the gravity is important.

Black holes formation in collisions of transplanckian particles is one of outstanding problems in theoretical physics. Our aim in this talk is to overview the current understanding of the problem.

Study of transplanckian collisions in gravity has a long history. In 80's-90's the problem has been discussed mainly in superstring theory frameworks [2, 3, 4, 5, 6, 7, 8, 9, 10] and was considered as an academical one, since the four dimensional Planck scale E_{Pl} is $\approx 10^{19}$ GeV, and energies satisfying $\sqrt{s} > E_{Pl}$ wholly out of reach of terrestrial experiments.

The situation has been changed after the proposal of TeV gravity scenario [1]. The D -dimensional Planck energy $E_{Pl,D}$ plays the fundamental role in TeV gravity, it has the electroweak scale of \sim TeV, as this would solve the hierarchy problem. TeV gravity is strong enough to play a role in elementary particle collisions at accessible energies.

The TeV gravity assumes the brane world scenario [11] that means that all light particles (except gravity) are confined to a brane with the 4-dimensional world sheet embedded in the D -dimensional bulk. The collider signatures of such braneworld scenarios would be energy non-conservation due to produced gravitons escaping into the bulk, signatures of new Kaluza-Klein particles as well as signatures of black hole (BH) [12, 13, 14, 15, 16] and more complicated objects such as wormholes (WH) formations [17, 18, 19] (see [20, 21] about WHs in astrophysics).

According the common current opinion the process of BH formation in transplanckian collision of particle may be adequately described using classical general relativity. We also believe that the same is true for the WH production [17]. Calculations based on classical general relativity support [22, 23] the simple geometrical cross section of black hole production in particles collisions, which is proportional to the area of the disk

$$\sigma = f\pi R_S^2(E), \quad (1)$$

where R_S is the Schwarzschild radius of the black hole formed in the particles scattering process and it is defined by the center-of-mass collision energy $E = \sqrt{s}$, and f is a formation factor of order unity. Colliding particles in hadron colliders are partons and the total cross section for black hole production is calculated using a factorization hypothesis in which the parton-level process is integrated over the parton density functions of the protons [24]. If the geometric cross section were true and colliding particles carry few TeV, the LHC would produce black holes at a rate ~ 1 Hz for $M_{Pl,D} = 1$ TeV, becoming a black hole factory [14, 16].

However, BH formation in particle collision is a threshold phenomena and the threshold is of order the Planck scale $M_{Pl,D}$ [25]. The exact value of the threshold is unknown since it depends of quantum gravity description of colliding particles. BH production rates depend on the value of $M_{Pl,D}$ [26]. Current bounds [27] are dimension-dependent but lie around $M_{Pl,D} \gtrsim 1$ TeV. Taking simple estimation for cross section

(1) with $f \sim 1$ above the threshold one can conclude that the cross section of semiclassical BHs production above the threshold at the LHC varies between 15 nb and 1 nb for the Planck scale between 1 TeV and 5 TeV. Note, that this cross section is compatible, for example, with $t\bar{t}$ production [28]. Just after production BHs quickly ($\sim 10^{-26}$ s) evaporate via Hawking radiation [29] with a characteristic temperature of ~ 100 GeV [14, 16]. However, since produced BHs are light they decay into only a few high energy particles and this would be difficult to disentangle from the background [30].

A natural question arises: can we catalyze the semiclassical process of the BHs formation and increase the production factor f in (1)? This is the main question that we are going to discuss in this talk. Let us note that in this talk we are going to deal with semiclassical consideration and make few notes of the region of its applicability. We will search for theoretical possibilities to increase the formation factor in the formula for the geometrical cross section. There are effects that work in the opposite direction and push the collision energy needed for BH formation considerably higher than $M_{Pl,D}$. These effects are related with the energy loss by colliding particles prior to the formation of the BH horizon [31] and the effects of the charge [32].

In fact there are few possibilities in our disposal to increase the formation factor. We are going to explore the following proposals

- find effects related with nontrivial dynamics of 3-brane embedding in D-dimensional space-time
- change the background (4-dimensional background or D-dimensional one), in particular, we can to add 4-dimensional the cosmological constant (or cosmological constant in D-dimensional space-time)
- take into account that shock wave in D-dimensional space-time can be made of from closed string excitations.

Some attempts toward these directions are presented in this talk.

In the last years numerous papers have been devoted to improvement calculations based on classical general relativity to get more precise estimates for the cross-section (1) [32, 37]. In particular, numerical calculations have been performed and they confirmed (1) and gave the estimations for the production factor [57]. The effects of finite size[34, 35], charge[32] and spin[38] have been considered. It has been found that the effects of mass, spin, charge and finite size of the incoming particles are rather small. The effects of the cosmological constant have been considered in papers[39, 40, 41, 42, 43, 44, 45, 46, 47]. In these papers estimation of the cross section of the BH production for particles colliding in $(A)dS$ backgrounds have been made. AdS case has been studied mainly within the AdS/CFT context, and dS case within possible cosmological applications [45, 47]. It has been found that the negative cosmological constant increases the cross section, meanwhile the positive cosmological constant works in the opposite direction destroying the trapped surface at the critical value of the cosmological constant and by this reason presumably holding up the BHs production.

Quantum field theory is a local theory in the Minkowsky space [48, 49]. However if we take into account effects of quantum gravity then some form of nonlocality is occurred. The problem of (non)locality in quantum gravity was addressed in [2, 50, 4], and more recently in [51]

The talk is organized as follows. In Sect. 2 we present a setup to study the BH formation in particle collisions. We follow [9] and present a natural generalization of this approach to the brane world case. We briefly summarize the main achievements of study the BH formation in flat background within the classical general gravity picture (trapped surfaces technics) and discuss why we can trust classical description.

In Sect.4. we discuss a physical picture of a black hole formation in the rest frame of one of colliding particles. This picture permits from simple calculations make a conclusion about the BH formation.

In Sect. 5 we generalize the above consideration to the case of the BH formation in (A)dS. This consideration presents a special interest because in contrast to the flat background in the AdS case there are no general theorems that guarantee the BH formation from the CTS formation. We discuss here an influence of structure of the shock wave on the BH formation. This structure is defined by gravitation interaction of matter fields. We also perform calculations of the cross section of the BH production in different cases and show agreements with trapped surfaces results. We conclude that the process of BH formation is catalyzed by a negative cosmological constant as well as by the string dilaton.

2 Setup. Milestones and Notations

2.1 D-dimensional Planckian Energy

In TeV gravity scenario we assume that all particles and fields (except gravity) are confined to a brane with 4-dimensional world sheet embedded in the D -dimensional bulk. Matter fields leave on the brane and do not feel extra dimensions, only gravity feels $n = D - 4$ extra dimensions, see Fig.1.

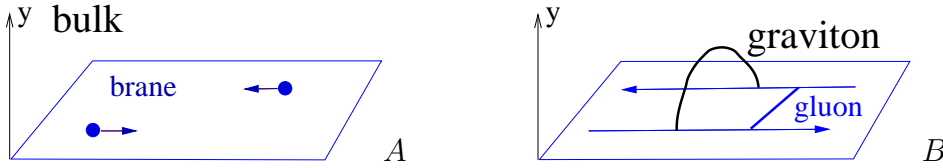


Figure 1: A. Colliding particles on the brane. B. D-dimensional graviton and 4-dimensional gluon exchanges.

According to the common current opinion the process of BH formation in transplanckian collision of particles, i.e. in regions where

$$\sqrt{s} \gg E_{Pl}, \quad (2)$$

may be adequately described using classical general relativity. We also believe that the same concerns the WH production. This is because in the transplanckian region (2) the de Broglie wavelength of a particle

$$l_B = \frac{\hbar c}{E} \quad (3)$$

is less than the Schwarzschild radius corresponding to this particle,

$$l_B << R_{S,D}, \quad (4)$$

here $R_{S,D}$ is the D-dimensional Schwarzschild radius in TeV gravity. In phenomenologically reasonable models with $n \geq 2$ the Schwarzschild radius corresponding to colliding particles with energies ≈ 1 TeV is $R_{S,D} \gtrsim 10^{-16} \text{ cm}$. In the usual 4-dimensional gravity the Schwarzschild radius corresponding to the same particles is of order $R_{S,4} \sim 10^{-49} \text{ cm}$, that is a negligible quantity comparing with the de Broglie wavelength of particles with energy about few TeV.

Although these types of processes are classical it is instructive to have a full picture starting from a general quantum field theory setup and pass explicitly to the classical

description of processes in question. This point of view is useful to deal with effects on the boundary of the classical applicability. By this reason we start in the next subsection from this general setup [9], and in Sect.2.? we present the brane extension of this approach [13].

2.2 Transition amplitudes and cross section of the BH/WH production

We start from quantum mechanical formula for the cross section σ_{AB} of a process

$$|A\rangle \Rightarrow |B\rangle. \quad (5)$$

To calculate this cross section we calculate the transition amplitude between these states

$$\langle A|B\rangle = \int \Psi_A^*(X_A, t) \mathcal{K}(X_A, t; X_B, t') \Psi_B(X_B, t') dX_A dX_B \quad (6)$$

where X are generalized coordinates, specifying the system, $\Psi_A(X, t)$, is a wave function of the state A including its asymptotical dynamics. The transition amplitude in the generalized coordinate representation is given by the Feynman integral. In our case we deal not only with particles but also with gravity. In particular, we discuss the process where the final state $|B\rangle$ is the state corresponding to the black hole. To this purpose we use a modification [9] of the standard formula [52]:

- For simplicity we work in $1+3$ formalism where spacetime is presented as a set of slices (more general formulation is described in [9]). At the initial time t we deal with a slice Σ and at the final time t' with a slice Σ' .
- Generalized coordinates include a metric g and matter fields ϕ .
- The state at on a initial time is specified by a three-metric h_{ij} and field ϕ and final state by a three-metric h'_{ij} and ϕ' .
- The transition amplitude in this generalized coordinate representation is given by Feynman integral [9]

$$\mathcal{K}(h, \phi, t; h', \phi', t') = \int e^{\frac{i}{\hbar} S[g, \phi]} \prod_{\substack{\phi|_{\tau=t} = \phi, \quad g|_{\tau=t} = h \\ \phi|_{\tau=t'} = \phi', \quad g|_{\tau=t'} = h'}} \mathcal{D}\phi(\tau) \mathcal{D}g(\tau) \quad (7)$$

where the integral is over all four-geometries and field configurations which match given values on two spacelike surfaces, Σ and Σ' and matter on them, $S[g, \phi]$ is the action. The integral in (7) includes also summation over different topologies.

- The transition amplitude given by the functional integral includes gauge fixing and Faddeev-Popov ghosts (all these are omitted in (7) for simplicity).

- We are interested in the process of a black hole creation in particles collisions. Therefore,
 - we specify the initial configuration h and ϕ on Σ without black holes, i.e. causal geodesics starting from Σ reach the future null infinity ¹;
 - we specify the final configuration h' and ϕ' on Σ' as describing black hole, i.e. Σ' contains a region from which the light does not reach the future null infinity ².

The explanation of notions used in above footnotes is given in [9], see also Appendix. For more details see [53, 54].

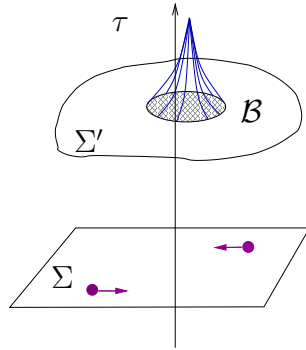


Figure 2: A slice Σ at $\tau = t$ is an initial slice with particles and a slice Σ' at $\tau = t'$ is a slice with a black hole \mathcal{B} . Null geodesics started from the shaded region do not reach null infinity.

In Figure 2 a slice with two colliding particles at $\tau = t$, and $\tau = t'$ with the BH area are presented. To describe such a process in the framework of a general approach (7) we have to find a classical solution of the Einstein equations with the matter, our moving particles, that corresponds to this picture, Figure 2, and then study quantum fluctuations. We do not have analytical solutions describing this process.

Finding of such solutions is a very difficult problem. It is solved only at low dimensional case, see [55, 56, 13] and refs. therein. In 4-dimensional case this problem has been solved numerically only recently by Choptiuk and Pretorius [57]. The solution, as it has been mentioned in Introduction, assumes a construction of a model for gravitational particles. We present this construction in the next subsection.

¹More precise, this condition means that Σ is a partial Cauchy surface with asymptotically simple past in a strongly asymptotically predictable space-time.

²This means that Σ' is a partial Cauchy surface containing black hole(s), i.e. $\Sigma' - J^-(\mathcal{I}^+)$ is non empty.

2.3 D-dimensional gravitational model of relativistic particles

To start a classical description of BH production in collision of elementary particles we need a gravitational model of relativistic particles. At large distances the gravitational field of particle is the usual Newtonian field. The simplest way to realize this is just to take the exterior of the Schwarzschild metric, i.e. in D-dimensional case away a particle we expect to have

$$ds^2 = \left(1 - \left(\frac{R_{S,D}}{R}\right)^{D-3}\right) dt^2 + \left(1 - \left(\frac{R_{S,D}}{R}\right)^{D-3}\right)^{-1} dR^2 + R^2 d\Omega_{D-2}^2, \quad (8)$$

where $R_{S,D}$ is the Schwarzschild radius

$$R_{S,D}^{D-3}(m) = \frac{16\pi G_D m}{(D-2)\Omega_{D-2}} = \frac{2m}{(D-2)\Omega_{D-2} M_{Pl,d}^{D-2}}, \quad (9)$$

here G_D is D-dimensional Newton gravitational constant, c the speed of light (in almost all formula we take $c = 1$) and Ω_{D-2} is the geometrical factor,

$$\Omega_{D-2} = \frac{2\pi^{(D-1)/2}}{\Gamma[(D-1)/2]}, \quad (10)$$

Γ is Euler's Gamma function. Here we present D-dimensional formula, in particular for $D = 4$, $R_{S,4}(m) = 2G_4 m$. We also use the expression of the Schwarzschild radius in term of the Planck mass, $R_{S,4}(m) = m/4\pi M_4^2 = m/\bar{M}_4^2$.

The interior of the Schwarzschild metric is supposed to fill with some matter. The simplest possibility is just to take a Tolman-Flories interior incompressible perfect fluid solution [58, 59]. As another model of relativistic particles one can consider a static spherical symmetric solitonic solution of gravity-matter equations of motion, the so-called boson stars (authors of refs.[60, 61] deal with 4-dimensional space-time, but it not a big deal to get D-dimensional extantions).

In the case of brane scenario few comments are in order. In the simplest brane models we deal with matter only on the brane and we do not have matter out of the brane to fill the interior of the D-dimensional Schwarzschild solution. However, in the string scenario³ there are closed string excitations which are suppose to be available in the bulk. One can assume that the matter in the bulk is a dilaton scalar field and deal with string inspired D-dimensinal generalization boson stars

$$ds^2 = \left(1 - \left(\frac{\mathcal{A}(R)}{R}\right)^{D-3}\right) dt^2 + \left(1 - \left(\frac{\mathcal{B}(R)}{R}\right)^{D-3}\right)^{-1} dR^2 + R^2 d\Omega^2, \quad (11)$$

where

$$\mathcal{A}(R) = A + A_1/R + \dots \quad (12)$$

Note, that stars in modified gravity and on branes have been considered in [62] and [63, 64], respectively.

³Open string excitations are located on brane, closed string excitations propagate on the bulk

2.4 Shock wave as a model of ultra relativistic moving particle

To consider ultra relativistic moving particle we have to make a boost of metric (11) with the large Lorentz boost factor $\gamma = 1/\sqrt{1 - v^2/c^2}$. The Schwarzschild sphere under this boost flattens up to an ellipsoid, see Figure 3.

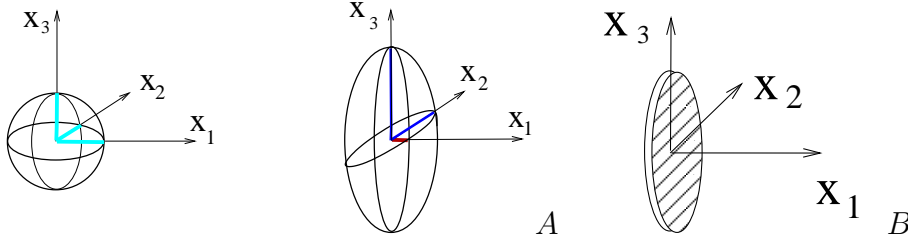


Figure 3: A. Flattening of the Schwarzschild sphere in the boosted coordinates. B. Schematic picture for the shock wave as a flat disk.

One can consider an approximation when γ is taken infinitely large and $E = \gamma A$ is fixed. The result metric is the Aichelburg-Sexl (AS) metric [65, 66], a gravitational shock wave, where the non-trivial geometry is confined to a $D - 2$ -dimensional plane traveling at the speed of light, with Minkowski spacetime on either side,

$$ds^2 = -2dUdV + dX_i^2 + F(X)\delta(U)dU^2, \quad i = 2, 3, \dots, D-1, \quad (13)$$

where $V = (X^0 + X^1)/\sqrt{2}$, $U = (X^0 - X^1)/\sqrt{2}$. The form of the profile of the shock wave F depends on the behavior of $\mathcal{A}(r)$.

In particular, in the infinite boost limit where we also take $m \rightarrow 0$ and hold p fixed, the metric (8) reduces to an exact shock wave metric (13) with the shape function F being the Green function of the $D - 2$ -dimensional Laplace equation

$$\Delta_{R^{D-2}} F = -\frac{2p\sqrt{2}}{M_{Pl}^{D-2}} \delta^{(D-2)}(X). \quad (14)$$

here $\delta^{(D-2)}(X) = \prod_{i=2}^{D-1} \delta(X^i)$,

$$F(X) = \frac{p2\sqrt{2}}{(D-4)\Omega M_{Pl,D}^{D-2}} \frac{1}{\rho^{D-4}} \quad (15)$$

where $\rho^2 = (X^2)^2 + \dots (X^{D-1})^2$. For $D=4$ the shape is

$$F(X) = -\frac{p\sqrt{2}}{\pi M_4^2} \ln \frac{\rho}{\varepsilon} \quad (16)$$

Note, that the metric (13) is obtained in the infinite boost limit when the source has zero rest mass. For fast particles of nonzero rest mass, the shock wave approximation

breaks down far away from the moving particle, more precisely at transverse distances from the source which are of the order of

$$\ell \sim r_h(m)/\sqrt{1-v^2}. \quad (17)$$

At these distances the field lines will spread out of the null transverse surface orthogonal to the direction of motion. But for $b \ll \ell$ one can use the shock wave field to extract the information about the black hole formation to the leading order in m/p . These shock waves are presented in Figure 4 as sphere flattened up to the disk. Two such shock waves, moving in opposite directions, see Figure 4.B give the pre-collision geometry of the spacetime. Though the geometry is not known to the future of the collision, since the shock wave solutions inevitably break down when the fields of different particles cross, at the moment of collision a trapped surface can be found [22, 67, 23].

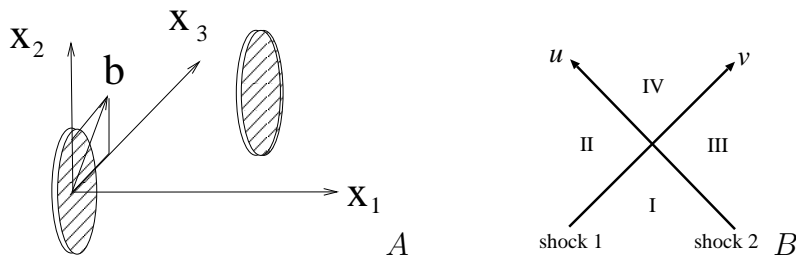


Figure 4: A. Ultra relativistic colliding particles in $D - 1$ -dimensional space; b is the $D - 2$ -dimensional impact vector. B. Ultra relativistic colliding particles in U, V -plane

According to [22, 67, 23], the trapped surfaces do form when $b \lesssim R_{S,D}$, and have the area of the order $\sim R_{S,D}^2$, where $R_{S,D}$ is the horizon radius given by (19) (see below).

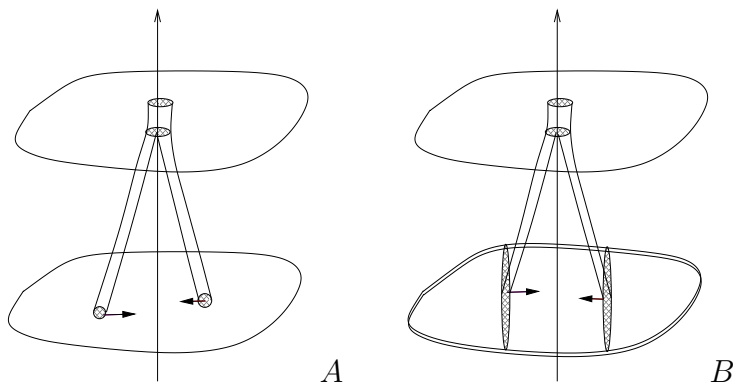


Figure 5: A. Colliding two stars; the initial space-time is asymptotically flat. B Colliding shock waves as models of ultra relativistic particles; the initial space time is not asymptotically flat.

Infinitely thin shock is an idealization. In reality the shocks will have a finite width w since γ is large but not infinite. The corresponding shocks have width $w_{class} \sim r/\gamma$, depending on the transverse distance r . Infinitely thin idealization leads to an appearance in the intersection of the planes of the two shock-waves a divergent curvature invariant [68]. In [69] has shown that this problem is an artifact of the unphysical classical point-particle limit and for a particle described by a quantum wavepacket, or for a continuous matter distribution, trapped surfaces indeed form in a controlled regime.

2.5 D-dimensional Thorne hoop conjecture and geometrical cross section

We expect to get the BH formation due to nonlinear interaction of gravitational fields produced by particles. The BH formation in classical general relativity is controlled by the Thorne hoop conjecture [70]. According the D-dimensional version of this conjecture if a total amount of matter mass M is compressed into a spherical region of radius R , a black hole will form if R is less than the corresponding Schwarzschild radius

$$R < R_{S,D}(M), \quad (18)$$

here $R_{S,D}(M)$ is given by (9).

In the case of ultra relativistic particle collisions the main argument for black hole formation is based on a modification of Thorne's hoop conjecture. According this modified conjecture if a total amount of energy E is compressed into a spherical region of radius R , a black hole will form if R is less than the corresponding Schwarzschild radius

$$R < R_{S,D}(E) \equiv \left(\Omega_n \frac{G_D E}{c^4} \right)^{\frac{1}{n+1}}, \quad (19)$$

Note that in this modified conjecture the horizon radius $R_{S,D}$ is set by the center-of-mass collision energy $E = \sqrt{s}$.

Few remarks are in order concerning this formulation. Literally speaking, as it is formulated above, it is not applicable in all situations. But this conjecture does applicable for two colliding particles. There are several calculations and arguments supporting this conjecture:

- One set of arguments is related with examining trapped surfaces formation in collisions of ultra relativistic particles [22, 16]. Note that commonly used evidence for black hole formation in collision of particles comes from the study of the collision of two Aichelburg-Sexl shock wave. This argument assumes that there is a solution interpolating between two shock waves and BH, Figure 5.B. However with this argument there is a problem that a space time with a shock wave is not asymptotically flat, that assumed in our scheme ⁴

⁴The AS metric also has a naked singularity at the origin. This is considered as an artifact of having used a black hole metric as the starting point, and assumed to be removed by taking a suitable mass distribution.

- The same problem is also with colliding plane wave [9]. An advantage to deal with plane waves is that in this case one can construct explicitly the metric in the interacting region.
- There is a non-trivial possibility to reduce the proof Thorne's hoop conjecture for ultra colliding relativistic particles to Thorne's hoop conjecture for slow moving relativistic particles (see Sect. 3 below)
- There are recent numerical calculations supporting (19) [57]. In [57] as a model of particles the boson star is taken [60]. Choptiuk and Pretorius have got a remarkable result that black holes do form at high velocities in boson star collisions and they found also that this happens already at a γ -factor of roughly one-third predicted by the hoop conjecture.

On the modified Thorne's hoop conjecture for ultra colliding relativistic particles the so-called geometrical cross section of BH production is based. It estimates the black hole production cross section by the horizon area of a black hole whose horizon radius $R_{S,D}$ is set by the center-of-mass collision energy $E = \sqrt{s}$, eq. (19). This estimation

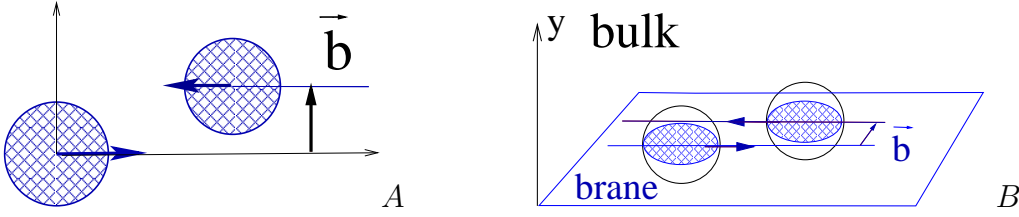


Figure 6: A. Colliding particles in $D - 1$ -dimensional space: b is $D - 2$ -dimensional impact vector and $\sigma \approx \mathcal{D}R_{S,D}^{D-2}$. B. Colliding particles on the 3-brane: 2-dimensional impact vector b and $\sigma \approx \pi R_{S,D}^2$. A shaded region indicates the projection of $D - 1$ dimensional Schwarzschild sphere onto the 3-brane

assumes that when the impact parameter b is smaller than $R_{S,D}$ then the probability of formation of a black hole is close to 1,

$$\sigma_{BH,D} \approx \mathcal{D}_{D-2} R_{S,D}^{D-2}(E), \quad (20)$$

\mathcal{D}_{D-2} is the volume of a plane cross section of the $D - 2$ dimensional unit sphere, see Figure 6.A where b is $D - 2$ -dimensional vector, i.e the area of of $D - 2$ -dimensional disk,

$$\mathcal{D}_D = \frac{\pi^{D/2}}{\Gamma(1 + \frac{D}{2})}; \quad (21)$$

In the 4-dimensional case this estimation gives

$$\sigma_{BH,4} \approx \pi R_{S,4}^2(E) \quad (22)$$

For the 3-brane embedding in the D-dimensional space-time we have

$$\sigma_{BH,3-brane} \approx \pi R_{S,D}^2(E) \quad (23)$$

since our particles are restricted on the 3-dimensional brane and the impact vector b is two dimensional vector, see Figure 6.B.

2.6 Looking from the rest frame of one of the incident particles

It is instructive to note that the similar analyze can be done in the in the rest frame of one of the incident particles [66]. This particle has large the de Broglie wavelength and has to be treated as a quantum particle. The gravitational field of the other, which is rapidly moving, looks like a gravitational shock wave, see Figure 7.

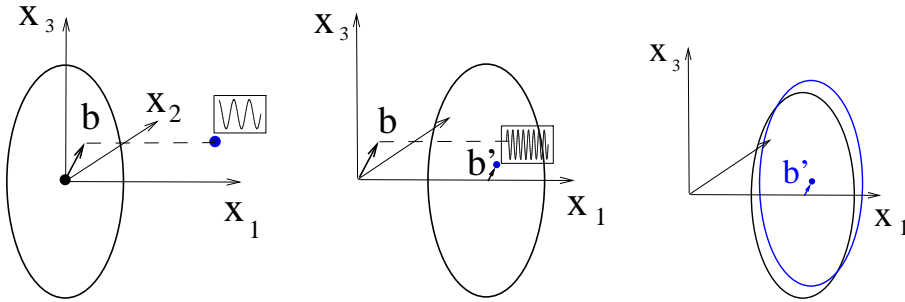


Figure 7: A. Ultra relativistic particle (shock wave) and a rest quantum particle, b is an impact vector. B. Quantum particle after a collision with the ultra relativistic particle, its impact vector b' just after collision decreases, $|b'| < |b|$ and its frequency increases. C. After collision particle which was in rest after collision move with an ultra relativistic velocity and looks as a shock wave.

Dynamics of the quantum particle can be described by a solution of the quantum Klein-Gordon equation in the shock wave background. This problem has been solved by 't Hooft [71]. Dynamics of the particle is given the eikonal approximation [72, 73] and is defined by the geodesics behavior near the shock wave. The approximation is valued for a large impact parameter. The shock wave focuses the geodesics down to a small impact parameter. Just in this region we expect the BH formation (see next section) and in this region the eikonal approximation is not nonapplicable. This give an explanation why a straightforward eikonal approximation does not describe the BH production. But it is instructive to see what the eikonal approximation can give and this is a subject of the next subsection.

The picture presented in Figure 7 is idealization. More precise approach would be started from one moving particle with γ rather large, but $\gamma \neq \infty$ and other particle in the rest. It should exist a classical solution that interpolates between this initial configuration and a configuration in the later time that represents two stars which are rather closed and move slowly respect each other. One can expect to estimate quantum fluctuations to such classical configuration.

2.7 BH formation and the eikonal approximation

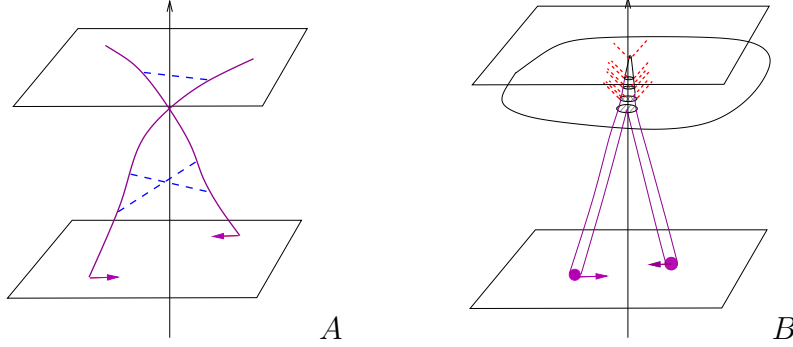


Figure 8: A. Ultra relativistic colliding particles with a large impact parameter. Blue lines represent the graviton exchange. B. Colliding particles with a small impact parameter and mass/energy enough to produce BH. Red dot lines represent BH evaporation.

For a large impact parameter in the transplanckian region one can use the eikonal approximation [72, 73]. Taking this approximation for the graviton exchange diagrams we get [74, 75, 76],

$$\mathcal{A}_{eik}(\mathbf{q}) = \mathcal{A}_{Born} + \mathcal{A}_{1-loop} + \dots = -8Ep \int d^2\mathbf{b} e^{-i\mathbf{q}\cdot\mathbf{b}} (e^{i\chi} - 1), \quad (24)$$

with the eikonal phase χ given by the Fourier transform of the Born amplitude in the transverse plane. The 4-dimensional Born amplitude for the graviton exchange is given by

$$\mathcal{A}_{Born}(\mathbf{q}) = \frac{2\pi G\gamma(s)}{Ep} \frac{1}{q_{\perp}^2 + \mu^2} \quad (25)$$

here μ is IR graviton mass regularization. The corresponding eikonal phase [74], is

$$\chi = \frac{2\pi G\gamma(s)}{Ep} \int \frac{d^2q_{\perp}}{(2\pi)^2} e^{i\mathbf{q}_{\perp}\cdot\mathbf{x}_{\perp}} \frac{1}{q_{\perp}^2 + \mu^2} = \frac{2\pi G\gamma(s)}{Ep} K_0(\mu b), \quad (26)$$

where $\gamma(s) = \frac{1}{2}((s - 2m^2)^2 - 2m^4)$, K_0 is the modified Bessel function.

For $b\mu \ll 1$ $K_0(\mu b) \sim \frac{1}{4\pi} \ln(\mu b)$ and we get the eikonal amplitude in term of Mandelstam variables

$$\mathcal{A}_{eik}(\mathbf{q}) = \frac{16\pi G\gamma(s)}{-t} \frac{\Gamma(1 - i\alpha(s))}{\Gamma(1 + i\alpha(s))} \left(\frac{4\mu^2}{-t} \right)^{-i\alpha(s)}, \quad \alpha(s) = \frac{2G\gamma(s)}{\sqrt{s(s - 4m^2)}} \quad (27)$$

The eikonal approximation with a real eikonal phase satisfies the unitarity condition

$$\sigma_{eik} = \frac{1}{16\pi^2 s^2} \int d^2q_{\perp} |\mathcal{A}_{eik}|^2 = \frac{\text{Im}\mathcal{A}_{eik}(0)}{s}. \quad (28)$$

and cannot describe the black hole formation.

However, the exact knowledge of $2 \rightarrow 2$ scattering amplitude can provide information about resonance states production. Therefore, if we know exact $2 \rightarrow 2$ scattering amplitude we expect to get an information about the BH formation due to the unitarity condition.

There is an analogy with BH production in higher energy and breather production in the 2 particles scattering in Sin-Gordon 2-dimensional model,

$$2 \text{ particles} \rightarrow \text{breather} \quad (29)$$

Indeed, the classical Sin-Gordon 2-dimensional model has so-called breather solutions with masses

$$m_n = \frac{16m}{\gamma} \sin \frac{n\gamma}{16}, \quad n = 1, \dots < \frac{8\pi}{\gamma} \quad (30)$$

The exact quantum $2 \rightarrow 2$ amplitude $\mathcal{A}_{\text{exact}}$ for massive particles in the 2-dimensional Sin-Gordon model [82] has an extra pole at

$$M^2 = 4m_1^2 - m_1^2 \left(\frac{\gamma}{8}\right)^2 + \dots \quad (31)$$

that is nothing but the pole corresponding to the first breather. One can see the breather contribution in the unitarity condition for amplitude of massive particles, $\mathcal{A}_{\text{exact}}$.

BH production in the collision of two particles can also seen as a violation of the unitary in the $2 \rightarrow 2$ elastic channel. Indeed, let us consider a scattering amplitude in two channels system,

$$\mathcal{A} = \begin{pmatrix} \mathcal{A}_{2p \rightarrow 2p} & \mathcal{A}_{2p \rightarrow BH} \\ \mathcal{A}_{BH \rightarrow 2p} & \mathcal{A}_{BH \rightarrow BH} \end{pmatrix} \quad (32)$$

$\mathcal{A}_{2p \rightarrow 2p}$ is the elastic scattering amplitude and $\mathcal{A}_{2p \rightarrow BH}$ is the inelastic one. The unitary condition means that

$$2\text{Im}\mathcal{A}_{2p \rightarrow 2p} = |\mathcal{A}_{2p \rightarrow 2p}|^2 + |\mathcal{A}_{2p \rightarrow BH}|^2 \quad (33)$$

So, if we expect $\mathcal{A}_{2p \rightarrow BH} \neq 0$ we have a violation of the the elastic unitarity,

$$2\text{Im}\mathcal{A}_{2p \rightarrow 2p} \neq |\mathcal{A}_{2p \rightarrow 2p}|^2 \quad (34)$$

The simple way to break unitarity is assume that in the eikonal approximation we deal with a complex eikonal phase. We expect the imaginary eikonal phase at small impact parameter and we write

$$\mathcal{A}_{\text{eik}}^{(2 \rightarrow 2)}(\mathbf{q}) = -2s \int_{|\mathbf{b}| > b_c} d^2\mathbf{b} e^{-i\mathbf{q} \cdot \mathbf{b}} (e^{i\chi} - 1) - 2s \int_{|\mathbf{b}| < b_c} d^2\mathbf{b} e^{-i\mathbf{q} \cdot \mathbf{b}} (e^{-\delta + i\chi} - 1), \quad (35)$$

$b_c \sim R_{S,4}$. We have

$$\sigma_{el} = \frac{1}{16\pi^2 s^2} \int \frac{d^2\mathbf{q}}{(2\pi)^2} |\mathcal{A}_{\text{eik}}^{(2 \rightarrow 2)}|$$

$$= 2 \int_{|\mathbf{b}| > b_c} d^2\mathbf{b} [1 - \cos \chi] + \int_{|\mathbf{b}| < b_c} d^2\mathbf{b} [1 + e^{-2\delta} - 2e^{-\delta} \cos \chi] \quad (36)$$

In accordance with the optical theorem,

$$\begin{aligned} \sigma_{total} &= \frac{1}{s} \text{Im} \mathcal{A}_{eik}^{(2 \rightarrow 2)}(0) \\ &= 2 \int_{|\mathbf{b}| > b_c} d^2\mathbf{b} [1 - \cos \chi] + 2 \int_{|\mathbf{b}| < b_c} d^2\mathbf{b} [1 - e^{-\delta} \cos \chi] \end{aligned} \quad (37)$$

and one can interpret the difference between (36) and (37) as a cross section of the BH production [76, 80, 81]

$$\sigma_{BH} = \sigma_{total} - \sigma_{el} = \int_{|\mathbf{b}| < b_c} d^2\mathbf{b} [1 - e^{-2\delta}] \quad (38)$$

To summarize the above discussion we can say that to describe the BH creation we would need to use the full classical solution describing the process, which however is difficult to handle. From other site, the full $2 \rightarrow 2$ particle amplitude would provide information about the BH production, but we are faraway from getting it. The elastic small-angle scattering amplitude given by eikonalized single-graviton exchange [71],[3],[77],[74], valued for large impact parameters $b \gg R_{S,4}$, cannot describe the BH formation. Computing the corrections in $b/R_{S,4}$ to the elastic scattering, one hopes to learn about the strong inelastic dynamics at $b \sim R_{S,4}$ [3],[78], [79].

2.8 Transition amplitudes and cross section for higher dimensional gravity and matter living on the brane

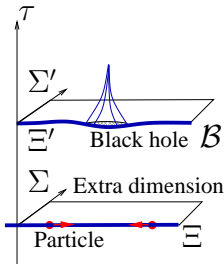
To consider the question of BH creation in high energy scattering in physical setting for low Planck scale we have to deal with two particles confined to the 3-brane which scatter due to the D -dimensional gravitational field, $D = 4 + n$, n is the number of large extra dimensions. For this purpose we have to make few modifications of formula (7) and take into account that particles interact with D -dimensional gravity and with matter living on the brane.

- For simplicity we work in $1 + n$ formalism. At the initial time t we deal with a slice Σ and at the final time t' with a slice Σ' . The slice Σ crosses the brane worldsheet over 3-dimensional slice Ξ and the slice Σ' crosses the brane worldsheet over 3-dimensional slice Ξ' .
- Generalized coordinate include D -dimensional metric g_{MN} and matter fields ϕ living on the brane B .
- The state at on a initial time is specified by a $3 + n$ -metric h_{IJ} on the slice Σ and fields ϕ on the slice Ξ and final state by a $3 + n$ -metric h'_{IJ} on the slice Σ' and ϕ' on the slice Ξ' .
- The transition amplitude in this generalized coordinate representation is given by Feynman integral, which is an extension of the formula from [9] to the brane world

$$\mathcal{K}(h, \phi, t; h', \phi', t') = \int e^{\frac{i}{\hbar} S[g, \phi]} \prod_{\substack{\phi|_{\tau=t} = \phi, \quad g|_{\tau=t} = h \\ \phi|_{\tau=t'} = \phi', \quad g|_{\tau=t'} = h'}} \mathcal{D}\phi(\tau, \vec{x}) \mathcal{D}g(\tau, \vec{X})$$

where the integral is over all $4 + n$ -geometries which match given values on two spacelike surfaces, Σ and Σ' and field configurations which match given values on two 3-dimensional spacelike surfaces, Ξ and Ξ'

- We specify the initial configuration h so that it corresponds to the Minkowski brane embedding in the bulk and matter fields ϕ on Ξ ;



A slice Σ' at $\tau = t'$ is a slice with a black hole \mathcal{B} . Null geodesics started from the shaded region on Ξ' do not reach null infinity.

A slice Σ at $\tau = t$ is an initial slice with with extra dimensions and particles living on the brane Ξ (blue thick line)

- We specify the final configuration h' on Σ' and ϕ' on Ξ' as describing black hole. \mathcal{B} is a in D-dimensional black hole.

It is not simple to find a solution with a D-dimensional black hole and a brane. The reason is that the usual D-dimensional black hole, say the Meyer-Perry black hole, solves the vacuum D-dimensional Einstein equation. But in the case of the presence of the brane the energy momentum tensor has an extra term providing the localization of the matter on the brane [83, 84, 85, 86], $T_{MN} \sim \delta(y)t_{MN}$; see detail discussions in papers [87, 88, 84, 89, 90, 91, 92, 93, 94, 95] and recent review [96]; note also the case of 1-brane in 1 + 2-space-time[13] and the case of 2-codimensional branes, where the problem can be solved for particular examples [97]).

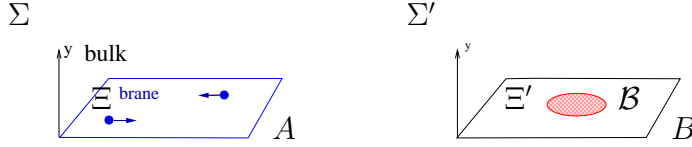


Figure 9: Slices with brane at different times: A. Initial slice Σ with brane Ξ and particles on the brane and without black holes; B. Finite slice Σ' with a black hole on the brane

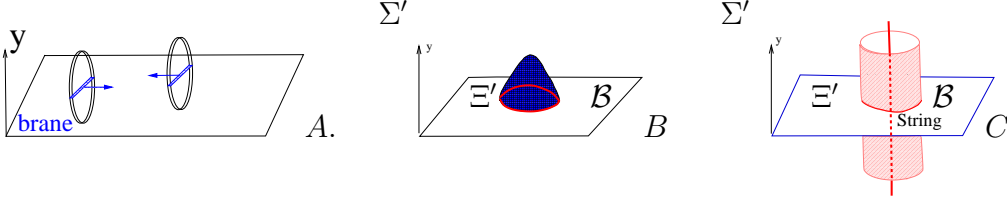


Figure 10: A: Ultra relativistic colliding particles on the 3-brane; a blue shaded region corresponds to a cross-section of the $D - 2$ dimensional disk by the 3-brane; B, C: Slices with brane at final time: B. Black hole with a source localized on a point at the brane Ξ' ; C. Black string with a source localized on a line along extra dimensions

3 Black hole formation in ultra relativistic particles collision as a classical gravitational collapse

3.1 Chargeless particles

We consider a collision of two massive particles with rest masses m and M , which move towards each other with relative velocity \vec{v} , and impact parameter b . Suppose that the particles in the rest frames are described by the Schwarzschild metric with the Schwarzschild radius, $R_{S,D}(m)$ and $R_{S,D}(M)$ given by (9).

For small relative velocity $v = |\vec{v}| \ll 1$, the cross section of the BH formation in the collision of these two BHs is of the order

$$\sigma \sim \mathcal{D}_{D-2} R_{S,D}^{D-2}(m), \quad (39)$$

where \mathcal{D}_{D-2} is the area of $D - 2$ -dimensional disk given by (21). Here we assume $M \sim m$. Estimation (39) is based on the Thorne hoop conjecture. This conjecture says that an apparent horizon forms if and only if matter with a mass M gets compressed enough such that the circumference in all directions satisfies the condition of $\mathcal{C} \lesssim 4\pi M$.

At large relative velocities, $v \rightarrow 1$, the cross section is different and is expected to be defined not by the rest masses but by the energy in the c.m.f., eq. (20). As has been mentioned in Sect.2.5 estimation (20) does not follow from the Thorne hoop conjecture.

Below we present an arguments in favor of (20) based on study of the system of two colliding particles in the rest frame of one of them. Our consideration follows main steps of Kaloper and Terning [99]. In this paper the authors considered 4-dimensional case and used the classical capture as a model of the black hole formation.

To show (20) following [66, 99] we go to the rest frame of one of two particles, say M . At large relative velocities, $v \rightarrow 1$ the gravitational field of the particle m is extremely strongly boosted in the rest frame of the particle M . In the infinite boost limit, where we also take $m \rightarrow 0$ and hold p fixed, the metric reduces to an exact shock wave metric [65, 66], given by (13) and (15). The metric around the shock wave is just two pieces of the flat space separated by the shock wave, and test particles move freely except when they cross the shock wave front. This picture is similar to the picture of the electric field lines of a highly boosted charge where the lines are compressed into the directions transverse to its motion [100]. Most of the scattering of a test particle takes place while it moves through this region with a more intense field and one can say that the shock wave behaves as a very thin gravitational lens.

Before the collision the particle M in its own rest frame stays at the point $X_0^1 = 0$, $X_0^2 = b$, $X_0^3 = 0$. We consider this particle as a test particle in the gravitation background (13) and therefore its movement after the collision is defined by the geodesics given by [66, 67, 101, 102]

$$V = V_0 + V_1 U + V_f \theta(U) + V_d \theta(U) U \quad (40)$$

$$X^i = X_0^i + X_1^i U + X_d^i \theta(U) U \quad (41)$$

with

$$V_f = \frac{1}{2}F, \quad X_d^i = \frac{1}{2}F_{,i}, \quad V_d = \frac{1}{2}F_{,i} \cdot X_1^i + \frac{1}{8}F_{,i}^2 \quad (42)$$

with the corresponding initial data. In X^0, X^1 coordinates this trajectory is

$$X_{(M)}^0(\tau) = \tau + \frac{F}{2\sqrt{2}}\theta(\tau) + \frac{F_{,i}^2}{16}\theta(\tau)\tau \quad (43)$$

$$X_{(M)}^1(\tau) = \frac{F}{2\sqrt{2}}\theta(\tau) + \frac{F_{,i}^2}{16}\theta(\tau)\tau \quad (44)$$

$$X_{(M)}^i(\tau) = b_i + \frac{1}{2\sqrt{2}}F_{,i}\theta(\tau)\tau, \quad (45)$$

here for simplicity we use $\tau = \sqrt{2}U$.

The m particle in the rest frame of the M particle moves along $U_{(m)}(\tau) = 0, X_{(m)}^i(\tau) = 0, i = 2, 3$. If the clocks for two particles are synchronized before the collision, i.e. $X_{(M)}^0(\tau) = X_{(m)}^0(\tau)$ for $\tau < 0$, we have

$$X_{(m)}^0(\tau) = X_{(m)}^1(\tau) = \tau + \frac{F}{2\sqrt{2}}\theta(\tau) + \frac{F_{,i}^2}{16}\theta(\tau)\tau \quad (46)$$

$$X_i = 0, \quad i \geq 2 \quad (47)$$

The distance between the M and m particles after the collision is given by

$$\mathcal{R}(\tau)^2 = (X_{(M)}^2(\tau))^2 + (X_{(M)}^1(\tau) - X_{(m)}^1(\tau))^2 = b^2(1 - \tau \frac{v_f}{b})^2 + \tau^2 \quad (48)$$

here

$$v_f = -F_{,2} \quad (49)$$

The minimal distance is achieved at $\tau = \tau_{min}$

$$\tau_{min} = \frac{b}{1 + v_f^2}v_f \quad (50)$$

and is given by the formula

$$\mathcal{R}_{min}(b) = \frac{b}{\sqrt{1 + v_f^2}}. \quad (51)$$

In a reasonable approximation

$$\mathcal{R}_{min}(b) \approx \frac{\pi M_{Pl,4}^2 b^2}{p} \quad (52)$$

The relative velocity of the m and M particles after the collision is

$$\vec{v}(\tau) = \left(\frac{\frac{d}{d\tau}(X_{(M)}^1(\tau) - X_{(m)}^1(\tau))}{\frac{d}{d\tau}(X_{(M)}^0(\tau))}, \frac{\frac{d}{d\tau}X_{(M)}^2(\tau)}{\frac{d}{d\tau}(X_{(M)}^0(\tau))}, 0 \right) = \left(\frac{-1}{1 + \frac{F_{,i}^2}{16}}, \frac{\frac{1}{2\sqrt{2}}F_{,2}}{1 + \frac{F_{,i}^2}{16}}, 0 \right) \quad (53)$$

Since $F \sim p$, for large values of p , the velocity \vec{v} has small components. Therefore after the collision, in the rest frame of the M particle we can use the non relativistic description and, in particular, apply the Thorne hoop conjecture. At this point our consideration is different from [99], where the capture process, related with the Laplace old idea [53], is interpreted as the BH production. In particular, we can say that if the minimal distance between particles after the collision less then the Schwarzschild radius of the M particle (in the rest frame) then the m particle would be captured by the M particle and we interpret this as a BH formation. The requirement that the minimal distance between particles is smaller than the Schwarzschild radius of the M particle,

$$R_{S,D}(M) > \mathcal{R}_{min}(b), \quad (54)$$

gives a restriction on the impact parameter

$$b < b_*, \quad b_*^2 = \frac{2s}{M_{Pl,4}^4} \quad (55)$$

here we use that $s = 2Mp$.

Hence for all b satisfying (55) the M particle will capture the m particle and interpreting this process as the black hole formation we get a cross section

$$\sigma = \pi b_*^2 = \frac{s}{M_{Pl,4}^4} \quad (56)$$

This answer is in agreement with estimations of the cross section based on the trapped surface area [23]. These estimations are based on the area theorem which states that the horizon area of the black hole must be greater than area of trapped surface, giving a lower bound on the mass of the black hole.

Comparing (55) with the restriction of the validity of the classical description,

$$\frac{1}{M_{Pl,4}} < b \lesssim \frac{\sqrt{s}}{M_{Pl,4}^2}, \quad (57)$$

we see that the above considerations are valid only for the transplanckian energies

$$s > M_{Pl,4}^2 \quad (58)$$

The right estimation in (57) also means a validity of the shock wave approximation $b \ll l$ with l given by (17), since the RHS of (17) is nothing but $p/8\pi M_4^2$.

The above calculations are essentially more simple than finding the trapped surface in the case of non head-on collision, and by this reason we call above estimation the "express-check" of BH formation.

3.2 Charged particles

It is instructive to perform the express-check of BH formation in the case of charged shock waves. These shock waves have been obtained by boosting the Reissner-Nortström for arbitrary D in spherical static (Schwarzschild) coordinates [103]

$$ds^2 = -g(R)dT^2 + g(R)^{-1}dR^2 + R^2 d\Omega_{D-2}^2, \quad (59)$$

$$g(R) = 1 - \left(\frac{R_{S,D}(m)}{R} \right)^{D-3} + \frac{Q^2}{R^{2(D-3)}}, \quad (60)$$

$R_{S,D}(m)$ is related with m by (9) and Q is related to charge q as follows:

$$Q^2 = \frac{8\pi G_D q^2}{(D-2)(D-3)}. \quad (61)$$

We note that this solution has meaning only for $R > R_{cl}$, where R_{cl} is the classical radius of the charge Q

$$R_{cl} = \left(\frac{\Omega_{D-2} q^2}{2(D-3)m} \right)^{1/(D-3)} \quad (62)$$

Now the Schwarzschild radius $R_{S,D}(m, Q)$ depends on the value of the charge and is the subjects of the equation

$$1 - \left(\frac{R_{S,D}(m)}{R_{S,D}(m, Q)} \right)^{D-3} + \frac{Q^2}{R_{S,D}(m, Q)^{2(D-3)}} = 0 \quad (63)$$

We note that this equation has solutions only for

$$Q^2 < Q_c^2, \quad Q_c^2 = \frac{1}{4} R_{S,D}^{2(D-3)}, \quad \text{or} \quad |q| \leq \frac{m}{\Omega_{D-2}} \sqrt{\frac{8\pi G_D (D-3)}{D-2}}. \quad (64)$$

For $Q^2 < Q_c^2$ there are two solutions

$$R_{S,D\pm}^{D-3}(m, Q) = \frac{R_{S,D}^{D-3}(m)}{2} \left(1 \pm \sqrt{1 - \frac{4Q^2}{R_{S,D}^{2(D-3)}(m)}} \right) \quad (65)$$

The first root is very small for small Q

$$R_{S,D-}^{D-3}(m, Q) \simeq \frac{Q^2}{R_{S,D}^{D-3}(m)} \quad (66)$$

and the second one we can consider as a small correction to the Schwarzschild radius of the m particle

$$R_{S,D+}(m, Q) \simeq R_{S,D}(m) \left(1 - \frac{1}{D-3} \frac{Q^2}{R_{S,D}^{2(D-3)}(m)} \right) \quad (67)$$

We see that for $D > 3$ the Schwarzschild radius decies then the charge Q^2 increases. The D -dimensional charged version of the Aichelburg-Sexl metric is [104]:

$$ds^2 = -2dUdV + dX_i^2 + F(|X|)\delta(U)dU^2, \quad (68)$$

$$F(\rho) = \begin{cases} -8G_4 p \ln \rho - \frac{2a_4}{\rho}, & (D=4), \\ \frac{16\pi G_D p}{(D-4)\Omega_{D-3}\rho^{D-4}} - \frac{2a_D}{(2D-7)\rho^{2D-7}}, & (D \geq 5), \end{cases} \quad (69)$$

where

$$a_D = \frac{2\pi(4\pi G_D p_e^2)(2D-5)!!}{(D-3)(2D-4)!!} \quad (D \geq 4) \quad (70)$$

Therefore, we get for $D = 4$

$$\mathcal{R}_{min}(b) \approx \frac{b}{v_{fQ}} = \frac{\pi M_4^2 b^2}{p} \left(1 + \frac{\pi C M_4^2 p_e^2}{pb}\right) \quad (71)$$

and for $D > 4$

$$\mathcal{R}_{min}(b) \approx \frac{b}{v_{fQ}} = \frac{\Omega_{D-3} b^{D-4}}{16\pi G_D p} \left(1 + \frac{32a_D \pi G_D p}{\Omega_{D-3} b^{3D-13}}\right) \quad (72)$$

As in the previous case, we can say that if the minimal distance between particles after the collision less then the Schwarzschild radius of the M particle, we have a BH formation. The requirement that the minimal distance between particles is smaller than the Schwarzschild radius of the M particle,

$$R_{S,D}(M, 0) > \mathcal{R}_{min}(b, Q), \quad (73)$$

gives a restriction on the impact parameter

$$b < b_{*Q}, \quad (74)$$

where b_{*Q} is a solution of the following equation for $D = 4$

$$R_{S,4}(m) = \frac{\pi M_4^2 b_{*Q}^2}{p} \left(1 + \frac{\pi C M_4^2 p_e^2}{pb_{*Q}}\right) \quad (75)$$

and for $D > 4$

$$R_{S,D}(m) = \frac{\Omega_{D-3} b_*^{D-4}}{16\pi G_D p} \left(1 + \frac{32a_D \pi G_D p}{\Omega_{D-3} b_*^{3D-13}}\right) \quad (76)$$

Hence for all b satisfying (74) we have a BH production. Writing b_{*Q} as

$$b_{*Q} = b_*(1 + Q^2 x), \quad (77)$$

we see that $x < 0$, i.e the cross section decreases. This result is in an agreement with the claim that charge effects reduce cross sections of the BH production [32].

3.3 Charged dilaton

The four-dimensional Maxwell-dilaton part of low-energy Lagrangian obtained from string theory is

$$S = \frac{1}{16\pi} \int d^4x \sqrt{-g} [R - 2(\nabla\phi)^2 - e^{-2a\phi} F_2^2], \quad a = 1 \quad (78)$$

here we consider a is an arbitrary parameter. This theory has static, spherically symmetric charged BH [105, 106]

$$ds^2 = -A^2(r)dt^2 + A^{-2}(r)dr^2 + R^2(r)d\Omega_2^2, \quad (79)$$

where

$$A^2(r) = \left(1 - \frac{r_+}{r}\right) \left(1 - \frac{r_-}{r}\right)^{(1-a^2)/(1+a^2)}, \quad (80)$$

$$R^2(r) = r^2 \left(1 - \frac{r_-}{r}\right)^{2a^2/(1+a^2)}. \quad (81)$$

and where r_+, r_- label the two free parameters. They are related to the physical mass and electric charge by

$$2M = r_+ + \frac{1-a^2}{1+a^2}r_-, \quad Q^2 = \frac{r_-r_+}{1+a^2}. \quad (82)$$

After γ -boost and rescaling $M = \gamma^{-1}p$, $Q^2 = \gamma^{-1}p_e^2$ one gets [107] the metric (13) with the following profile

$$F = - \left\{ 4p \ln \rho^2 + \frac{3-4a^2}{2(1-a^2)} \frac{\pi p_e^2}{\rho} \right\} \quad (83)$$

From (83) we see that increasing/decreasing of the cross section is defined by the the sign of the the effective charge

$$\alpha_{eff} = \frac{3-4a^2}{2(1-a^2)}, \quad (84)$$

that is negative for $\sqrt{3}/2 < a < 1$, see Figure 11.

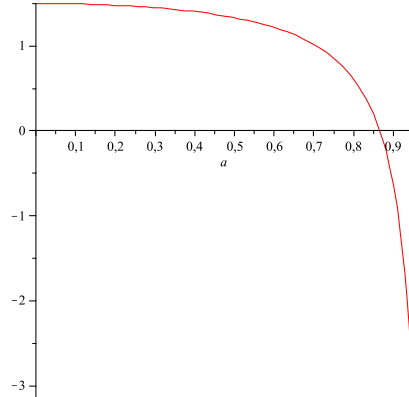


Figure 11: Effective charge for the dilaton shock wave

Therefore for this region we have negative values for α_{eff} and as result of get a catalyze of the BH production.

4 Black hole formation in ultra relativistic particle collisions in (A)dS as a classical gravitational collapse

4.1 Shock wave approximation of ultra relativistic particles in (A)dS

We consider a collision of two massive particles with rest masses m and M , which move towards each other in (A)dS background. We suppose that the particles in the rest frames are described by the exterior Schwarzschild (anti) de Sitter metric

$$ds^2 = -g(R)dT^2 + g(R)^{-1}dR^2 + R^2d\Omega_{D-2}^2, \quad (85)$$

$$g(R) = 1 - \left(\frac{R_{S,D}}{R}\right)^{D-3} - \epsilon \frac{R^2}{a^2}, \quad (86)$$

where $\epsilon = +1$ for dS and $\epsilon = -1$ for AdS, $R_{S,D} = R_{S,D}(m)$ is related to mass m as in (9). Now the Schwarzschild radius $R_h(m, a)$ depends on the value of the cosmological radius and is the subject of the equation

$$1 - \left(\frac{R_{S,D}(m)}{R_{S,D}(m, a)}\right)^{D-3} - \epsilon \frac{R_{S,D}^2(m, a)}{a^2} = 0 \quad (87)$$

We note the for $\epsilon = -1$ this equation always has a solution. For $a \gg m$ we have

$$R_{SAdS,D}(m, a) = R_{S,D}(m) \left(1 - \frac{1}{D-3} \frac{R_{S,D}^2(m)}{a^2}\right) \quad (88)$$

and we see that for $D > 3$ the Schwarzschild radius increases then the cosmological radius a^2 decresies.

In the dS case either there are two solutions of (63) or there is no solutions at all. The minimal cosmological radius a^2 below which there are no solution is

$$a_c^2 = c_D R_{S,D}^2(m), \quad (89)$$

where $c_D = (D-1)/(D-3)((D-1)/2)^{1/(D-3)}$ and in particular, $c_4 = \frac{27}{4}$.

For $a \gg a_c$ the minimal root of equation (63) with $\epsilon = 1$ is

$$R_{SdS,D}(m, a) = R_{S,D}(m) \left(1 + \frac{1}{D-3} \frac{R_{S,D}^2(m)}{a^2}\right) \quad (90)$$

When these two particles move towards each other in the (A)dS space-time with small cosmological constants and with a small velocity, the cross section of the BH formation in this collision is of order $\sigma \sim \pi R_{S(A)dS,D}^{D-2}(m)$ and as in the flat case are just negligible numbers.

We can increase the cross section of the BH formation by increasing up to ultra-relativistic the relative velocity of the colliding particles. To describe this collision in

the (A)dS backgrounds in is convenient to describe in In terms of the dependent plane coordinates, $U, V, X^2 \dots X^{D-1}, X^D$, $\vec{X} = (X^2, \dots, X^{D-1})$, satisfying

$$-2UV + X_2^2 + \dots + X_{D-1}^2 + \epsilon X_D^2 = \epsilon a^2 \quad (91)$$

As in the flat case to consider the collision of two particles with large relative velocities, $v \rightarrow 1$, we consider this process from the rest frame of one of these two particles, say the M particle. The gravitational field of the m particle is extremely strongly boosted. In this limit, we can approximate the field by the linearized Schwarzschild (A)dS metric, boosted to a very large velocity. In the infinity boost limit, where we also take $m \rightarrow 0$ and hold p fixed, the Schwarzschild (A)dS metric reduces to a shock metric [108, 90, 109, 110, 111, 112]. In terms of the dependent plane coordinates, the line element of the shock wave space-time is

$$ds^2 = -2dU dV + d\vec{X}^2 + \epsilon dX_D^2 + F(\vec{X})\delta(U)dU^2. \quad (92)$$

The shock wave shape function F is a fundamental solution of the equation

$$\left(\Delta_{D-2} + \epsilon \frac{D-2}{a^2} \right) F = -16\sqrt{2}\pi G_D \bar{p} \delta(\vec{n}, \vec{n}_0), \quad (93)$$

where Δ_{D-2} is the Laplace–Beltrami operator on a $(D-2)$ -dimensional sphere \mathbb{S}^{D-2} in the dS case or on a $(D-2)$ -dimensional hyperboloid \mathbb{S}^{D-2} in the AdS case, $\vec{n} = \vec{x}/|\vec{x}|$, \vec{n}_0 is the location of the particle on the sphere or the hyperboloid, \bar{p} is the energy of the shock wave, and G_D is the D -dimensional gravitational constant.

For $D = 4$, $\epsilon = 1$ we deal with

$$F_4^{dS} = p \left(-1 + \frac{Z_4^0}{2a} \ln \frac{a + Z_4^0}{a - Z_4^0} \right) \quad (94)$$

where p is related with p_{ABG} from [45], $p = 8\sqrt{2}p_{ABG} = 8\sqrt{2}\bar{p}_{ABG}G_4$.

For $D = 5$, $\epsilon = 1$

$$F_5^{dS}(\xi) = \frac{3\sqrt{2}\pi p_5}{a} \frac{2\xi^2 - 1}{\sqrt{1 - \xi^2}} \quad (95)$$

where $\xi = Z_5/a$ and $p_5 = \bar{p}G_5$.

For $D = 4$, $\epsilon = -1$ we deal with the shock wave in the AdS space-time

$$F_4^{AdS} = p \left(-1 + \frac{Z_4^0}{2a} \ln \frac{a + Z_4^0}{-a + Z_4^0} \right) \quad (96)$$

A relation with notations in [46] is $p = 4\sqrt{2}p_{AB} = 4\sqrt{2}\bar{p}_{AB}G_4$.

For $D = 5$, $\epsilon = -1$

$$F_5^{AdS}(\bar{M}, Z_5) = -\frac{p}{a} \left(\frac{1 - \frac{2Z_5^2}{a^2}}{\sqrt{\frac{Z_5^2}{a^2} - 1}} + \frac{2Z_5}{a} \right) \quad (97)$$

and $p = 3\pi\bar{M}_{ABJ}$, [47].

4.2 Geometrical Picture of BH production in Particle Collision in (A)dS

The metric around the shock wave (92) is just two pieces of (A)dS space separated by the shock wave, and test particles move along timelike geodesics of (A)dS space. From the point of view of the flat dependent coordinates the particles move freely except when they cross the shock wave front. This picture is similar to the picture in the flat case and the shock wave behaves as a very thin gravitational lens. The amount of bending of the geodesics crossing the shock is proportional to p . Thus for $\gamma \rightarrow \infty$ we can get scattering at almost right angle in the $U - X^1$ plane, and a particle with a large impact parameter after crossing the shock wave passes very close to the path of the boosted particle. If the scattered particles end up within a distance smaller than the horizon of the target particle then we get a black hole formation.

Let us show this explicitly. We consider the geodesics starting from the point $Z_4^0 \neq 0, Z_2^0 \neq 0, Z_3^0 = 0$, and use the parametrization

$$AdS : \quad Z_4^0 = a \cosh \vartheta, \quad Z_2^0 = a \sinh \vartheta, \quad (98)$$

$$dS : \quad Z_4^0 = a \cos \vartheta, \quad Z_2^0 = a \sin \vartheta. \quad (99)$$

For these initial data for timelike geodesics we have [113, 45]:

$$V(U) = V^0 S(U) + V_1 U + B \Theta(U) S(U) + C \Theta(U) U, \quad (100)$$

$$Z_2(U) = Z_2^0 S(U) + A_2 \Theta(U) U, \quad (101)$$

$$Z_3(U) = 0, \quad (102)$$

$$Z_4(U) = Z_4^0 S(U) + A_4 \Theta(U) U, \quad (103)$$

where

$$\begin{aligned} A_i &= -\frac{1}{6} \Lambda Z_i^0 G(0), \quad i = 2, 3, \quad A_4 = \frac{1}{2} \left[\epsilon F_{,4}(0) - \frac{1}{3} \Lambda Z_4^0 G(0) \right], \\ B &= \frac{1}{2} F(0), \quad C = \frac{1}{8} \left[\epsilon F_{,4}^2(0) + \frac{1}{3} \Lambda F^2(0) - \frac{1}{3} \Lambda (Z_4^0 F_{,4}(0))^2 \right], \end{aligned} \quad (104)$$

and

$$F(0) \equiv F(Z_4^0), \quad G = Z_p F_{,p} - F \quad (105)$$

$$S(U) = \sqrt{1 + \frac{1}{3} \Lambda (\dot{U}^0)^{-2} U^2}, \quad V_1 = \dot{V}^0 / \dot{U}^0 \quad (106)$$

From the above formula we can estimate the minimal distance between the particles. In term of X^0, X^1 coordinate for the M particle we have the following coordinates

$$Z_{(M)}^0 = \frac{V+U}{\sqrt{2}} = \frac{V^0}{\sqrt{2}} S(U) + \frac{V_1+1}{\sqrt{2}} U + \Theta(U) \frac{B S(U) + CU}{\sqrt{2}} \quad (107)$$

$$Z_{(M)}^1 = \frac{V-U}{\sqrt{2}} = \frac{V^0}{\sqrt{2}} S(U) + \frac{V_1-1}{\sqrt{2}} U + \Theta(U) \frac{B S(U) + CU}{\sqrt{2}} \quad (108)$$

$$Z_{(M)}^2(U) = Z_2^0 S(U) + A_2 \Theta(U) U, \quad (109)$$

$$Z_{(M)}^4(U) = Z_4^0 S(U) + A_4 \Theta(U) U, \quad (110)$$

$$Z_{(M)}^3(U) = 0 \quad (111)$$

Since in the coordinate system we deal with the times are synchronized we suppose that

$$Z_{(m)}^0 = Z_{(M)}^0 \quad (112)$$

For the m particle we have

$$Z_{(m)}^0 = \frac{V+U}{\sqrt{2}} = \frac{V^0}{\sqrt{2}} S(U) + \frac{V_1+1}{\sqrt{2}} U + \Theta(U) \frac{B S(U) + CU}{\sqrt{2}} \quad (113)$$

$$Z_{(m)}^1 = Z_{(m)}^0, \quad (114)$$

$$Z_{(m)}^4 = a \quad (115)$$

$$Z_{(m)}^i = 0, i = 2, 3 \quad (116)$$

The distance is

$$\mathcal{R}_{(A)dS}^2((M), (m), U) = (Z_{(m)}^1 - Z_{(M)}^1)^2 + (Z_{(m)}^2 - Z_{(M)}^2)^2 + (Z_{(m)}^3 - Z_{(M)}^3)^2 \quad (117)$$

Taking into account that $Z_{(m)}^1 = Z_{(m)}^0$ and $Z_{(m)}^0 = Z_{(M)}^0$ we get

$$\mathcal{R}_{(A)dS}^2((M), (m), U) = 2U^2 + Z_2^{0^2} \left(\sqrt{1 + \frac{\epsilon}{a^2} U^2} + \frac{A_2}{Z_2^0} U \right)^2 \quad (118)$$

Comparing this answer with the similar answer in the flat background, we see that their are similar, except the factor $S(U)$ given by (106) instead of the unit.

For dS_4 case

$$\mathcal{R}_{dS}^2(\tau) = \tau^2 + b^2 \left(\sqrt{1 + \frac{\tau^2}{2a^2}} - \frac{B_2}{b} \tau \right)^2, \quad (119)$$

where

$$B_2 = \frac{p}{2b\sqrt{2}}, \quad b = a \sin \vartheta, \quad (120)$$

$R_{dS}^2(\tau)$ as a function of τ has one minimum at the real point τ_{min} .

The expression for τ_{min} can be expanded on $1/a^2$

$$\tau_{min} \approx b \frac{B_2}{B_2^2 + 1} + b_0 \frac{b_0^2}{4a^2} \frac{B_2(B_2^2 - 2)}{(B_2^2 + 1)^3} + \mathcal{O}(1/a^4) \quad (121)$$

(here we assume that $b > 0$). Note that in the leading order

$$\tau_{min} \approx b \frac{B_2}{B_2^2 + 1} \mathcal{O}(1/a^2) \quad (122)$$

that is in agreement with (50)

Under assumption $B_2 \gg 1$ we get

$$\tau_{min,dS_4} = \frac{2\sqrt{2}b^2}{p} + \frac{2b^5}{a^2p^2} \quad (123)$$

and, therefore

$$\mathcal{R}_{dS_4,min}(b) \approx \frac{2\sqrt{2}b^2}{p} \left(1 + \frac{2b^4}{a^2p^2}\right) \quad (124)$$

For AdS_4 case

$$\mathcal{R}_{AdS_4}^2(\tau) = \tau^2 + b^2 \left(\sqrt{1 - \frac{\tau^2}{2a^2}} - \frac{B_2}{b} \tau \right)^2 \quad (125)$$

and

$$B_2 = \frac{p}{2b\sqrt{2}}, \quad b = a \sinh \vartheta, \quad (126)$$

As for dS case, under assumption $B_2 \gg 1$ we get

$$\tau_{min,AdS_4} = \frac{2\sqrt{2}b^2}{p} - \frac{2b^5}{a^2p^2} \quad (127)$$

and, therefore

$$\mathcal{R}_{AdS_4,min}(b) \approx \frac{2\sqrt{2}b^2}{p} \left(1 - \frac{2b^4}{a^2p^2}\right) \quad (128)$$

Comparing (52), (128) and (124) we see that for the same values of b^2/p the value of $R_{min}(b^2/p)$ is minimal for the AdS case.

If the scattered particles end up within a distance smaller than the horizon of the target particle then we certainly expect a black hole to form. Therefore, to estimate the geometrical cross section we find all impact parameters for which takes place the condition

$$R_{S,D}(M) > R_{min}(b), \quad (129)$$

where $R_{S,D}(M)$ is the Schwarzschild radius (without the cosmological constant). Taking into account estimations of the minimal distances (124) and (128) we get

$$R_{S,D}(M) > \frac{2\sqrt{2}b^2}{p} \left(1 + \epsilon \frac{2b^4}{a^2p^2}\right) \quad (130)$$

At the limit $a \rightarrow \infty$ we have

$$2MG_4 > \frac{2\sqrt{2}b^2}{p}, \quad (131)$$

and this relation holds for all $b^2 < b_0^2 = MG_4 p / \sqrt{2}$,

We see that for the dS case the LHS of (130) as a function of b is an increasing function. In the case of the AdS it is increasing up to $b_{max} = (a^2 p^2 / 6)^{1/4}$. Therefore, restriction (130) holds for all $b < b_*$, where the critical value b_* satisfies the relation

$$R_{S,4}(M) = \frac{2\sqrt{2}b_*^2}{p} \left(1 + \epsilon \frac{2b_*^4}{a^2 p^2}\right) \quad (132)$$

and in the AdS case we also assume that $b_* < b_{max}$.

For the AdS_5 case

$$\mathcal{R}_{min} \approx \frac{b}{B_2} \left(1 - \frac{b^2}{4a^2 B_2^2}\right) \quad (133)$$

Taking into account that in 5-dim case

$$B_2 = \frac{p}{2\sqrt{2}b^2}, \quad (134)$$

we get

$$\mathcal{R}_{min,5} \approx \frac{b}{B_2} \left(1 - \frac{b^2}{4a^2 B_2^2}\right) = \frac{2\sqrt{2}b^3}{p} \left(1 - \frac{2b^6}{a^2 p^2}\right) \quad (135)$$

and from this formula we see that cross section in AdS case more then the crossection in the flat case for the same value of p . Therefore the negative cosmological constant catalyzes the BH production.

5 Conclusion

We rederive the classical geometrical cross section of the BH production reconsidering the process of two transplanckian particles collision in the rest frame of one of incident particles. This consideration permits to use the standard Thorne's hoop conjecture for a matter compressed into a region to prove a variant of the Thorne's hoop conjecture dealing with a total amount of compressed energy in the case of colliding particles.

We show that the process of BH formation is catalyzed by the negative cosmological constant and by a special scalar matter. In opposite, it is relaxed by the positive cosmological constant and at a critical value just turns off. Also we note that the cross section is sensible to the compactification of extra dimensions and particular brane models and this will be studied in separated paper in details.

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A BH as an initial/final data

Let (\mathcal{M}, g) is the space-time with a metric, \mathcal{M} is a manifold.

Black holes are conventionally defined in asymptotically flat space-times by the existence of an event horizon H .

The horizon H is the boundary of the causal past of future null infinity, i.e. it is the boundary of the set of events in space-time from which one can escape to infinity in the future direction.

To be more precise we need few definitions[53, 54].

All notions work for BH production in the particle collision as well as for gravitational collapse.

A.1 (Weakly) Asymptotically Simple Space-time

A (oriented in time and space) space-time $(\mathcal{M}, g_{\mu\nu})$ is *asymptotically simple* ([53], p.246) if there exists a smooth manifold $\tilde{\mathcal{M}}$ with metric $\tilde{g}_{\mu\nu}$, boundary \mathcal{I} , and a scalar function Ω regular everywhere on $\tilde{\mathcal{M}}$ such that

- $\tilde{\mathcal{M}} - \mathcal{I}$ is conformal to \mathcal{M} with $\tilde{g}_{\mu\nu} = \Omega^2 g_{\mu\nu}$,
- $\Omega > 0$ in $\tilde{\mathcal{M}} - \mathcal{I}$ and $\Omega = 0$ on \mathcal{I} with $\nabla_\mu \Omega \neq 0$ on \mathcal{I} ,
- Every null geodesic on $\tilde{\mathcal{M}}$ contains, if maximally extended, two end points on \mathcal{I} .

\mathcal{I} consists of two disjoint pieces \mathcal{I}^+ (future null infinity) and \mathcal{I}^- (past null infinity) each topologically $R \times S^2$,

$$\mathcal{I} = \mathcal{I}^+ \cup \mathcal{I}^- \quad (136)$$

One can symbolically write

$$\tilde{\mathcal{M}} = \mathcal{M} \cup \partial\mathcal{M}, \quad (137)$$

where $\partial\mathcal{M} = \mathcal{I}$.

If M satisfies the Einstein vacuum equations near \mathcal{I} then \mathcal{I} is null.

A space-time \mathcal{M} is *weakly asymptotically simple* if there exists an asymptotically simple M_0 with corresponding \tilde{M}_0 such that for some open subset K of \tilde{M}_0 including \mathcal{I} , the region $\mathcal{M}_0 \cup K$ is isometric to an open subset of \mathcal{M} . This allows \mathcal{M} to have more infinities than just \mathcal{I} .

A.2 Asymptotically Flat Space-time

A space-time is *asymptotically flat* if it is *weakly asymptotically simple and empty*, that is, near future and past null infinities it has a conformal structure like that of Minkowski space-time.

A.3 Global (Partial) Cauchy surface

Let \mathcal{S} be a space-like hypersurface. The future(past) domain of dependence of \mathcal{S} , denoted $D^+(\mathcal{S})(D^-(\mathcal{S}))$, is defined by [53]

$$D^\pm(\mathcal{S}) = \{p \in \mathcal{M} | \text{Every past(future) inextendible causal curve through } p \text{ intersect } \mathcal{S}\} \quad (138)$$

The full domain of dependence of \mathcal{S} is defined as

$$D(\mathcal{S}) = D^+(\mathcal{S}) \cup D^-(\mathcal{S}) \quad (139)$$

The set \mathcal{S} for which $D(\mathcal{S}) = \mathcal{M}$ is called a Cauchy surface, or a *global Cauchy surface*. Causal curves cross the global Cauchy surface just one time.

The space-like hypersurface \mathcal{S} is called the *partial Cauchy surface* if non one causal curve crosses it more then one time [53], see Figure 12.

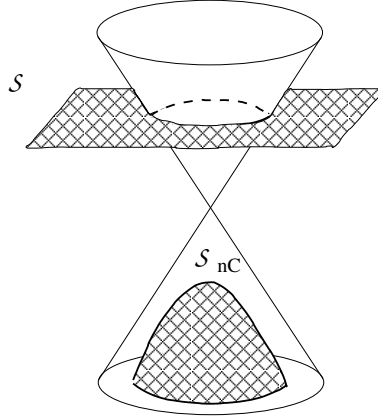


Figure 12: A. The Minkowski space time M^4 with the Cauchy surface \mathcal{S} and a surface \mathcal{S}_{nC} which is not the Cauchy surface, [53], Fig.13

A.4 Causal Future $J^+(p)$ and Causal Past $J^-(p)$ of the Point

A causal future $J^+(p)$ (past $J^-(p)$) of the point p is defined as

$$J^\pm(p) = \{q \in \mathcal{M}, \text{ such that there is future (past) oriented causal curve } \gamma(\tau), \text{ so that } \gamma(0) = p, \gamma(1) = q\} \quad (140)$$

A causal future $J^+(\Sigma)$ of the surface Σ is defined as

$$J^-(\Sigma) = \{q \in \mathcal{M}, \text{ if there is future oriented causal curve } \gamma(\tau), \text{ so that } \gamma(0) = p \in \Sigma, \gamma(1) = q\} \quad (141)$$

In particular,

$$J^-(\mathcal{I}^+) = \{q \in \mathcal{M}, \text{ if there is future oriented} \quad (142)$$

$$\text{causal curve } \gamma(\tau), \text{ so that } \gamma(0) = p \in \mathcal{I}^+, \gamma(1) = q\}$$

A.5 Black Holes in Asymptotically Flat Space-time

Black holes are conventionally defined in asymptotically flat space-times by the existence of an event horizon H . The horizon H is the boundary $\dot{J}^-(\mathcal{I}^+)$ of the causal past $J^-(\mathcal{I}^+)$ of future null infinity \mathcal{I}^+ , i.e. it is the boundary of the set of events in space-time from which one can escape to infinity in the future direction.

The *black hole region* B is

$$B = M - J^-(\mathcal{I}^+)$$

and the *event horizon*

$$H = \dot{J}^-(\mathcal{I}^+).$$

A.6 Future (strongly) Asymptotically Predictable Space-time

A space-time is *future asymptotically predictable* if there is a surface \mathcal{S} in spacetime that serves as a Cauchy surface for a region extending to future null infinity⁵.

This means that there are no "naked singularities" (a singularity that can be seen from infinity) to the future of the surface \mathcal{S} .

Let Σ be a partial Cauchy surface in a weakly asymptotically simple and empty space-time (M, g) . The space-time (M, g) is (future) *asymptotically predictable from* Σ if \mathcal{I}^+ is contained in the closure of $D^+(\Sigma)$ in \tilde{M}_0 .

If, also, $J^+(\Sigma) \cap \bar{J}^-(\mathcal{I}^+, \bar{M})$ is contained in $D^+(\Sigma)$ then the space-time (M, g) is called *strongly asymptotically predictable* from Σ . In such a space there exist a family $\Sigma(\tau)$, $0 < \tau < \infty$, of spacelike surfaces homeomorphic to Σ which cover $D^+(\Sigma) - \Sigma$ and intersects \mathcal{I}^+ . One could regard them as surfaces of constant time.

A.7 Black Hole on a Surface

A *black hole on the surface* $\Sigma(\tau)$ is a connected component of the set

$$B(\tau) = \Sigma(\tau) - J^-(\mathcal{I}^+, \bar{M}).$$

One is interested primarily in black holes which form from an initially non-singular state. Such a state can be described by using the partial Cauchy surface Σ which has an *asymptotically simple past*, i.e. the causal past $J^-(\Sigma)$ is isometric to the region $J^-(\mathcal{I})$ of some asymptotically simple and empty space-time with a Cauchy surface \mathcal{I} . Then Σ has the topology R^3 .

⁵This notion gives a formulation of Penrose's cosmic censorship conjecture.

A.8 Boundary conditions in the path integral representation

One is interested primarily in black holes which form from an initially non-singular state. Such a state can be described by using the partial Cauchy surface Σ which has an *asymptotically simple past*, i.e. the causal past $J^-(\Sigma)$ is isometric to the region $J^-(\mathcal{I})$ of some asymptotically simple and empty space-time with a Cauchy surface \mathcal{I} . Then Σ has the topology R^3 .

In the path integral representation considered (6) we suppose that we deal with a set of space-times $\{(M, g_{\mu\nu})\}$ which are weakly asymptotically simple and empty and strongly asymptotically predictable. We take into account only such $(M, g_{\mu\nu})$ that contains Σ' and Σ'' so that

- Σ' is a partial Cauchy surface with asymptotically simple past, $\Sigma' \sim R^3$.
- $\Sigma'' = \Sigma(\tau'')$ contains a black hole, i.e. $\Sigma'' - J^-(\mathcal{I}^+, \bar{M})$ is nonempty.

In particular, if one has the condition $\Sigma' \cap \bar{J}^-(\mathcal{I})$ is homeomorphic to R^3 (an open set with compact closure) then Σ'' also satisfies this condition.

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